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FRACTIONAL q-CALCULUS AND NEW GENERALIZATION OF GENERALIZED M-SERIES

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ABSTRACT
This paper is devoted to fractional q-derivative of special functions. To begin with the theorem on term by term q-fractional differentiation has been derived. Fractional q-differentiation of new generalization of Generalized M-series has been obtained.

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INTRODUCTION

q-Analogue of Differential Operator

Al-Salam [3], has given the q-analogue of differential operator as

$$D_q f(x) = \frac{f(xq) - f(x)}{x(q-1)} \tag{1.1}$$

This is an inverse of the q-integral operator defined as

$$\int_x^\infty f(t)\, d(t:q) = x(1-q) \sum_{k=1}^{\infty} q^{-k} f(xq^{-k}) \tag{1.2}$$

WHERE $0 < |q| < 1$

FRACTIONAL Q-DERIVATIVE OF ORDER α:

THE FRACTIONAL Q-DERIVATIVE OF ORDER α IS DEFINED AS

$$\boldsymbol{D_{x,q}^{\alpha} f(x) = \frac{1}{\Gamma_q(-\alpha)} \int_0^x (x - yq)_{-\alpha-1} f(y) d(y; q)} \tag{1.2.1}$$

WHERE RE $(\alpha) < 0$

AS A PARTICULAR CASE OF (1.2.1), WE HAVE

$$\boldsymbol{D_{x,q}^{\alpha} x^{\mu-1} = \frac{\Gamma_q(\mu)}{\Gamma_q(\mu - \alpha)} x^{\mu-\alpha-1}} \tag{1.2.2}$$

The New Generalization of Generalized M-Series

Here , first the notation and the definition of the New **Generalization of** Generalized M-series, introduced by Ahmad Faraj , Tariq Salim , Safaa Sadek, Jamal Ismai [5] has been given as

$$\underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(a_1 ..., a_p; b_1, ..., b_q; z) = \underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(\mathrm{z}),$$

$$\underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(z)=\sum_{k=0}^{\infty}\frac{(a_1)_{km}\dots(a_p)_{km}}{(b_1)_{kn}\dots(b_q)_{kn}}\;\frac{z^k}{\Gamma(\alpha k+\beta)}$$

…(1.3.1)

Here $\alpha,\beta\in C, \mathrm{Re}(\alpha)>0,\ \mathrm{Re}(\beta)>0$,$(a_j)_{km}$, $(b_j)_{kn}$ are the pochammer symbols and m,n are non-negative real numbers.

MAIN RESULTS

IN THIS SECTION, WE DRIVE THE RESULTS ON TERM BY TERM Q-FRACTIONAL DIFFERENTIATION OF A POWER SERIES. AS PARTICULAR CASE WE WILL THE FRACTIONAL Q-DIFFERENTIATION OF NEW GENERALIZATION OF GENERALIZED M-SERIES.

THEOREM 1: IF THE SERIES $\underset{p,q;m,n}{\overset{\alpha,\beta}{M}}(z)$ converges absolutely for $|q|<\rho$ THEN

$$D_{z,q}^{\mu}\left\{z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a_1)_{km}\dots(a_p)_{km}}{(b_1)_{kn}\dots(b_q)_{kn}}\;\frac{z^k}{\Gamma(\alpha k+\beta)}\right\}$$

$$=\sum_{k=0}^{\infty}\frac{(a_1)_{km}\dots(a_p)_{km}}{(b_1)_{kn}\dots(b_q)_{kn}}\;\frac{1}{\Gamma(\alpha k+\beta)}D_{z,q}^{\mu}z^{k+\lambda-1} \qquad (2.1)$$

Where $\mathrm{Re}(\lambda)>0,\ \mathrm{Re}(\mu)<0,\ 0<|q|<1$

PROOF: STARTING FROM THE LEFT SIDE AND USING EQUATION (1.2.1), WE HAVE

$$D_{z,q}^{\mu}\left\{z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a_1)_{km}\dots(a_p)_{km}}{(b_1)_{kn}\dots(b_q)_{kn}}\;\frac{z^k}{\Gamma(\alpha k+\beta)}\right\}$$

$$=\frac{1}{\Gamma_q(-\mu)}\int_0^z(z-yq)_{-\mu-1}\,y^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a_1)_{km}\dots(a_p)_{km}}{(b_1)_{kn}\dots(b_q)_{kn}}\;\frac{z^k}{\Gamma(\alpha k+\beta)}\,d(y;q)$$

$$=\frac{z^{\lambda-\mu-1}}{\Gamma_q(-\mu)}\int_0^1(1-tq)_{-\mu-1}\,z^{\lambda-1}\sum_{k=0}^{\infty}\frac{(a_1)_{km}\dots(a_p)_{km}}{(b_1)_{kn}\dots(b_q)_{kn}}\;\frac{z^k}{\Gamma(\alpha k+\beta)}\,d(t;q) \qquad (2.2)$$

NOW THE FOLLOWING OBSERVATION ARE MADE

(i) $\sum_{k=0}^{\infty}\frac{(a_1)_{km}\dots(a_p)_{km}}{(b_1)_{kn}\dots(b_q)_{kn}}\;\frac{z^k}{\Gamma(\alpha k+\beta)}$ converges absolutely and therefore uniformly on domain of x over the region of integration.

(ii) $\int_0^1\left|(1-tq)_{-\mu-1}t^{\lambda-1}\right|\,d(t;q)$ IS CONVERGENT,

PROVIDED $\mathrm{Re}(\lambda)>0,\ \mathrm{Re}(\mu)<0,\ 0<|q|<1$

THEREFORE THE ORDER OF INTEGRATION AND SUMMATION CAN BE INTERCHANGED IN (2.2) TO OBTAIN.

$$=\frac{z^{\lambda-\mu-1}}{\Gamma_q(-\mu)}\sum_{k=0}^{\infty}\frac{(a_1)_{km}\dots(a_p)_{km}}{(b_1)_{kn}\dots(b_q)_{kn}}\;\frac{z^k}{\Gamma(\alpha k+\beta)}\int_0^1(1-tq)_{-\mu-1}\,t^{\lambda+k-1}d(t;q)$$

$$= \sum_{k=0}^{\infty} \frac{(a_1)_{km} \dots (a_p)_{km}}{(b_1)_{kn} \dots (b_q)_{kn}} \frac{z^k}{\Gamma(\alpha k + \beta)} \boldsymbol{D}_{\boldsymbol{z},\boldsymbol{q}}^{\boldsymbol{\mu}} z^{(n+\gamma)\alpha-\beta-1+\lambda-1}$$

Hence the statement (2.1) is proved.

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